A STRETCHED COORDINATE TECHNIQUE FOR NUMERICAL ABSORPTION OF EVANESCENT AND PROPAGATING WAVES IN PLANAR WAVEGUIDING STRUCTURES

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ABSTRACT

Berenger's PML technique is modified to allow for the absorbtion of evanescent waves, as well as propagating waves, in FDTD modeling of wave propagation in planar waveguiding structures. Analytical results are used to illustrate the validity and capability of the proposed modification. Results from FDTD and compact 2D-FDTD simulations demonstrate its performance.

INTRODUCTION

Berenger's perfectly matched layer (PML) approach to the truncation of FDTD grids has been used successfully in conjunction with electromagnetic radiation and scattering problems in two and three dimensions [1-3]. An alternative approach to the construction of the PMLs, directly from Maxwell's equations using stretched coordinates, was presented also in [4]. However, up to this point there has been no detailed investigation of the performance of PMLs with regards to absorption of bound waves with evanescent behavior transverse to their direction of propagation, of the type encountered in practice in conjunction with various types of planar waveguiding structures. Except from a brief discussion in [3], there has been no careful investigation of the effect that a PML, positioned parallel to the media interface that supports such bound waves, might have on the propagation characteristics of the wave, especially when the PML is brought close to the interface.

Published results from FDTD simulations of radiation and scattering problems indicate that the PMLs can be brought very close (as close as 2 grid cells) to the radiator/scatterer. Such a capability is highly desirable for the FDTD analysis of integrated microwave, millimeter wave and optical circuits, in order to keep computer memory requirements and CPU time at a minimum without sacrificing the numerical accuracy of the simulations. Such circuits include a variety of resonant structures, the electromagnetic behavior of which is dependent on the dispersive propagation characteristics of the waves supported by the waveguiding sections that comprise them. Consequently, it is important to investigate and comprehend the impact of PMLs on the dispersive characteristics of planar waveguiding structures before any circuit-level FDTD simulations using PMLs can be performed with confidence.

A first attempt to this investigation is presented in this paper. Through an analytic study of the eigenvalue problem of wave propagation in a dielectric slab waveguide with PMLs present on either side of the slab, we demonstrate that Berenger's PML disturbs the efective index of the propagating wave. Furthermore, we introduce a modification to the original PML that is shown to alleviate this difficulty. FDTD simulations of wave propagation in slab waveguides illustrate the numerical implementation of the modified PML. Finally, applications of the modified PML in compact 2D-FDTD simulations of open microstrip structures are used to demonstrate its accuracy.

THEORY

Using the stretched coordinate approach proposed in [4], Maxwell's curl equations are written as

$$\frac{\partial}{\partial t}\epsilon\vec{E} = \nabla_s \times \vec{H} \quad (1)$$

$$-\frac{\partial}{\partial t}\mu\vec{H} = \nabla_s \times \vec{E} \quad (2)$$

where

$$\nabla_s = \hat{x}\frac{1}{s_x}\frac{\partial}{\partial x} + \hat{y}\frac{1}{s_y}\frac{\partial}{\partial y} + \hat{z}\frac{1}{s_z}\frac{\partial}{\partial z} \quad (3)$$

We restrict our attention to the two-dimensional case with $\partial/\partial y = 0$. As it is well known, Maxwell's equations decouple into two independent sets, one involving

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the field components H_y, E_x, and E_z (TM polarization), and one involving the field components E_y, H_x, H_z (TE polarization). The pertinent equations for the TM polarization are

$$
\begin{aligned}
j\omega\epsilon E_{z_x} &= \frac{1}{s_x}\frac{\partial}{\partial x}(H_{y_x}+H_{y_z}) \\
&= \frac{1}{s_x}\frac{\partial}{\partial x}H_y \qquad (4)\\
j\omega\epsilon E_{x_z} &= -\frac{1}{s_z}\frac{\partial}{\partial z}(H_{y_x}+H_{y_z}) \\
&= -\frac{1}{s_z}\frac{\partial}{\partial z}H_y \qquad (5)\\
j\omega\mu H_{y_x} &= \frac{1}{s_x}\frac{\partial}{\partial x}E_{z_x} \qquad (6)\\
j\omega\mu H_{y_z} &= -\frac{1}{s_z}\frac{\partial}{\partial z}E_{x_z} \qquad (7)
\end{aligned}
$$

where the split-component formalism used with Berenger's PML condition, H_{y_x}, H_{y_z}, E_{x_z} and E_{z_x}, has been used. In order to derive the wave equation for the TM polarization, we substitute Eq. (4) in Eq. (6) and Eq. (5) in Eq. (7), giving

$$
\begin{aligned}
-\omega^2\mu\epsilon H_{y_x} &= \frac{1}{s_x}\frac{\partial}{\partial x}\frac{1}{s_x}\frac{\partial}{\partial x}H_y \qquad (8)\\
-\omega^2\mu\epsilon H_{y_z} &= \frac{1}{s_z}\frac{\partial}{\partial z}\frac{1}{s_z}\frac{\partial}{\partial z}H_y \qquad (9)
\end{aligned}
$$

Since $(H_{y_x}+H_{y_z}) = H_y$ we obtain

$$
-\omega^2\mu\epsilon H_y = \frac{1}{s_x}\frac{\partial}{\partial x}\frac{1}{s_x}\frac{\partial}{\partial x}H_y + \frac{1}{s_z}\frac{\partial}{\partial z}\frac{1}{s_z}\frac{\partial}{\partial z}H_y \qquad (10)
$$

Next, we consider the eigenvalue problem associated with the TM-modes of the slab dielectric waveguide shown in Figure 1. Nonmagnetic materials are assumed. The waveguide has thickness $2w$, constant permittivity ϵ_1, and its axis coincides with the z axis. Due to the symmetry of the structure, only the top half is shown. A slab of thickness l and permittivity $\epsilon_2 < \epsilon_1$ represents the medium outside the guide. Notice that $s_x = s_z = s = 1$ for both the guide and the adjacent slab. Beyond the ϵ_2 slab we introduced an M-layer structure terminated by a perfectly conducting plane. All layers in the structure have the same permittivity, ϵ_2; however, their thicknesses, d_i, and the values of s_{x_i}, are allowed to vary, while $s_{z_i} = 1$. Let β be the unknown propagation constant for even TM-mode propagation in z. A straightforward analysis of the eigenvalue problem results in the following eigenvalue equation

$$
\tan(k_{x_1}w) = \frac{k_{x_2}\epsilon_1}{k_{x_1}\epsilon_2} \times \mathcal{E} \qquad (11)
$$

where $\mathcal{E}$ is

$$
\mathcal{E} = \frac{1+\exp(-2k_{x_2}l)\exp(-2k_{x_2}\sum_{i=1}^{M} s_{x_i}d_i)}{1-\exp(-2k_{x_2}l)\exp(-2k_{x_2}\sum_{i=1}^{M} s_{x_i}d_i)} \qquad (12)
$$

where $k_{x_1}^2 = \omega^2\mu_0\epsilon_1 - \beta^2$ and $k_{x_2}^2 = \beta^2 - \omega^2\mu_0\epsilon_2$. For propagating modes, k_{x_2} is real and positive. At this point, it is appropriate to recall that the eigenvalue equation for the even TM modes for the case of the standard slab waveguide of thickness $2w$ and permittivity ϵ_1 embedded in a homogeneous medium ϵ_2 is given by $\tan(k_1 w) = (k_2\epsilon_1)/(k_1\epsilon_2)$. Thus, the term $\mathcal{E}$ of Eq. (12) may be thought of as the "error" term caused by the truncated domain and the presence of the layered structure. It can be seen that as $l \to \infty$ in Eq. (12), $\mathcal{E} \to 1$, recovering the eigenvalue equation for the standard slab waveguide. Eq. (12) also suggests that instead of having $l \to \infty$ the eigenvalue equation for the standard waveguide can be reclaimed by letting s_{x_i} assume large values. The value of s_{x_i} required is dependent on the problem. Further, we notice that s_{x_i} needs to be positive to cause the error terms to become negligible compared to 1. At this point, it is appropriate to recall that for Berenger's PML s_{x_i} is of the form $s_{x_i} = 1 - j(\sigma_i/\omega\epsilon_i)$ and, as such, it does not cause any additional attenuation (since k_{x_2} is real) to help reduce the error term in Eq. (11).

NUMERICAL IMPLEMENTATION

The analysis of the previous section suggests the use of s values in the stretched-coordinate formulation of the PML with $Re\{s\} > 1$ in order to facilitate rapid absorption of evanescent waves without affecting their propagation characteristics. Therefore, in order to encompass the general case of planar structures that have both radiating and waveguiding properties, we allow s to assume complex values with real part greater than 1. More specifically, s_i takes the form

$$
\begin{aligned}
s_i &= s_i' - js_i'' \\
&= s_i'\left(1 - j\frac{s_i''}{s_i'}\right) \\
&= s_i'\left(1 - j\frac{\sigma_i}{\omega\epsilon}\right) \qquad (13)
\end{aligned}
$$

where $i = x, y, z$. Notice that in addition to the real part of s_i, s_i', being greater than 1, the ratio σ_i/ϵ is scaled by s_i' also. With this notation for s_i, the modified Maxwell's system for the TM case takes the form

$$
\begin{aligned}
\left(\epsilon\frac{\partial}{\partial t} + \sigma_x\right)E_z &= \frac{1}{s_x'}\frac{\partial}{\partial x}(H_{y_x}+H_{y_z}) \qquad (14)\\
\left(\epsilon\frac{\partial}{\partial t} + \sigma_z\right)E_x &= -\frac{1}{s_z'}\frac{\partial}{\partial z}(H_{y_x}+H_{y_z}) \qquad (15)
\end{aligned}
$$

$$\left(\mu\frac{\partial}{\partial t}+\frac{\sigma_x\mu}{\epsilon}\right)H_{y_x} = \frac{1}{s'_x}\frac{\partial}{\partial x}E_z \quad (16)$$

$$\left(\mu\frac{\partial}{\partial t}+\frac{\sigma_z\mu}{\epsilon}\right)H_{y_z} = -\frac{1}{s'_z}\frac{\partial}{\partial z}E_x \quad (17)$$

RESULTS

As an example, consider a guide with a core of $w = 1.0$ cm and and index of refraction of 1.3, surrounded by air. Fig. 2 shows the analytical results of Eq. (11) and numerical results for $\lambda_0 = 3.0$ cm, $l = 0.5$ cm, with a 10 cell, parabolically graded PML with each layer being 1.0 mm thick. The solid line is the result of Eq. (11) with the dotted line being the result when $\mathcal{E} = 1$. The stars are obtained from the numerical implementation of Eqs. (14)-(17). The real part of s_{x_i} is varied in the PML as

$$s_{x_i} = s_{avg}\left(\frac{i}{10}\right)^2 + 1 \qquad i = 1, 2, \ldots, 10 \quad (18)$$

and the imaginary part is set equal to zero.

From Fig. 2 we see that the trend of the numerical implementation matches that of the analytical results. Further, in all cases the percent difference between the analytical and numerical results is less than 0.5%. We note that the magnitude of the normalized modal field at the PML interface is around 0.46, showing that it is not necessary to let the field decay significantly before introducing the PML.

A very useful application of this modified PML truncation scheme is in the compact 2D-FDTD dispersion analysis of open multiconductor waveguiding systems [6]. For lossless systems, (i.e. planar waveguides that do not support leaky modes) the field behavior in the transverse plane is totally evanescent. Therefore, we can remove the σ terms and simply use six field components as in standard FDTD.

As an example, Fig. 3 depicts the frequency dependence of the effective dielectric constant for the even and odd modes of the coupled microstrip shown in the insert obtained from a compact 2D-FDTD simulation with modified PML truncation. For this specific geometry, $w/h = 1$, $s/h = 2$, and $\epsilon_1 = 9.7\epsilon_0$. For the odd mode, perfect electric conductor (PEC) walls are placed on the left and bottom sides while 10 cell PML layers ($s_{avg} = 10$) are placed on the top and right sides. The even mode is modeled in a similar fashion, except that the PEC on the left wall is replaced with a perfect magnetic conductor (PMC). Even though the PML layers are placed only 5 cells from the conductor, the agreement with the results in [5] ($\triangle$) are accurate to within 1.25%.

In order to evaluate the effectiveness of the modified PML layer, a comparison was made to the case where the PML layer is removed. Fifteen cells from the conductor, a PEC is placed on the top side and a PMC is placed on the right side. Fig. 4 shows the power spectrum densities (PSD) of the transformed time domain responses for the odd mode with $\beta = 1.5(1/cm)$. The solid and dashed lines denote the spectrum obtained by using the modified PML and PEC/PMC boundary conditions respectively. The shift in frequency for the PEC/PMC boundary condition translates to a 8% error in ϵ_{eff}. Also note that the PSD for this case shows appreciable energy beyond the fundamental mode. This is due to excitation of higher order (waveguide) modes in the structure caused by the presence of the PEC/PMC walls.

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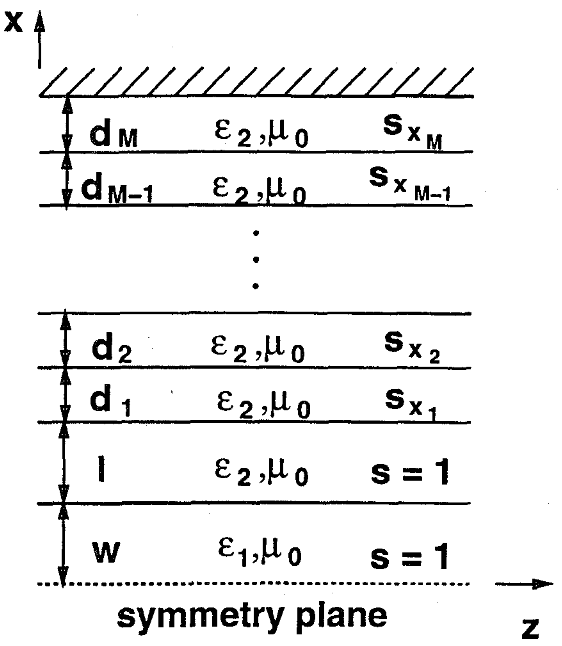

Figure 1: Slab waveguide geometry for analytically soluble problem.

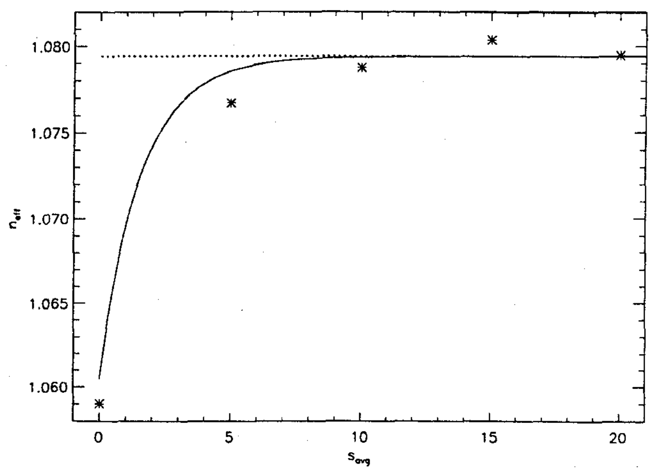

Figure 2: Results for example problem (— solution of Eq. (11), - - - solution of Eq. (11) with $\mathcal{E} = 1$, $*$ FDTD results).

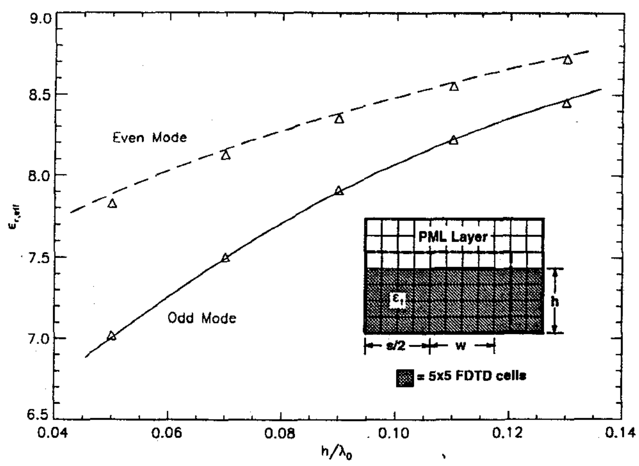

Figure 3: Effective permittivity of even and odd mode for a coupled microstrip (— and - - - 2D-FDTD results, $\triangle$ results from [5]).

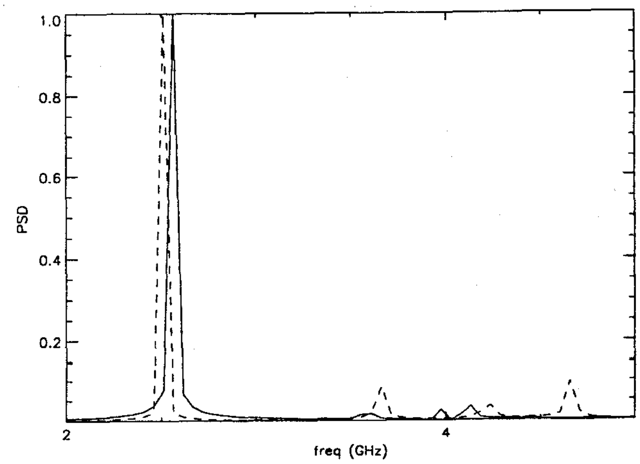

Figure 4: PSD of odd mode (— PML boundary condition, - - - PMC/PEC boundary condition).